Homework 1 Mathematical Prerequisites

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I pledge my honor that I have abided by the Stevens Honor System.

Sets:

1: f(x) = 0 → {0, 10, 20, 30, …}

2: a) A ∪ B → {0, 2, 4, 6, 8, 10, 12, …}

b) A ∩ B → {0, 10, 20, 30, …}

c) A \ B → {ø}

Functions:

1.

i. Injective, 0 not mapped to.

ii. Surjective, x=0, x=5 map to 2.

iii. Surjective, x=0, x=10 map to 2.

iv. bijective

2.

i. Injective, 1 not mapped to.

ii. Surjective, x=0, x=5 map to 0.

iii. Surjective, x=0, x=5 map to 0.

iv. Injective, 1 not mapped to.

3.

i. Injective, 1 not mapped to.

ii. Surjective, x=0, x=5 map to 0.

iii. Surjective, x=0, x=10 map to 0.

iv. Injective, 1 not mapped to.

Boolean Logic

1: A↔(B∧C)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | **(B∧C)** | **A↔(B∧C)** |
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

2. (A∨B) → C

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | **(A∨B)** | **(A∨B)→C** |
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | T | T |
| T | F | F | T | F |
| F | T | T | T | T |
| F | T | F | T | F |
| F | F | T | F | T |
| F | F | F | F | T |

3. A → (B ⊕ C)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | **(B ⊕ C)** | **A → (B ⊕ C)** |
| T | T | T | F | F |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | T | F | T | T |
| F | F | T | T | T |
| F | F | F | F | T |

Strings and Languages:

1. ∑ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, \*, /, (, ) }

2. “2+34/” ; “((01-\*/()” ; “03/0”

3. a. No number’s first digit is 0.

b. Does not end in an operator.

c. All open parentheses have a closing parenthesis.

d. No set of parentheses is empty.

e. No double operators.

f. No division by 0.

Proofs:

1. Proof by Contrapositive:

a) Use truth tables to prove (A → B) ↔ (¬B → ¬A)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | ¬A | ¬B | A→B | ¬B→¬A |
| T | T | F | F | T | T |
| T | F | F | T | T | T |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

b) State contrapositive of the following: “If is odd, then n is also odd.”

If n is not odd, then is not odd.

c) Use arithmetic to prove the contrapositive statement.

is not odd if n is not odd

Therefore, if is not odd, then is also not odd.

2. Proof by Induction: Given , prove that , for all .

Base Case:

Inductive Hypothesis:

Inductive Case:

Ind. Hyp.

arith.

arith.

Therefore: